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On the difference between the pole and the $\overline{\text{MS}}$ masses of the top quark at the electroweak scale

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Abstract

We argue that for a Higgs boson mass $M_H \sim 125$ GeV, as estimated from recent Higgs searches at the LHC, the inclusion of the electroweak radiative corrections in the relationship between the pole and $\overline{\text{MS}}$ masses of the top quark reduces the difference to about 1 GeV. This fact is relevant for the scheme dependence of electroweak observables as well as for the extraction of the top quark mass from experimental data.

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1 Introduction

For the precise understanding of the relationship between running and pole masses of particles, within the framework of the Standard Model (SM) of electroweak (EW) and strong interactions, it is mandatory to use the full SM Renormalization Group (RG) equations. In this paper, we focus in particular on the top quark mass. In published results, commonly only the QCD corrections are applied, however, also the corresponding EW corrections are important. Here we discuss the EW contributions to the SM RG equations and the related matching conditions and their numerical significance for the pole mass. The relevant corrections have been derived for the top quark in Refs. [1, 2]. Assuming the particle recently discovered at the CERN LHC [3, 4] to be the SM Higgs boson, it is possible to specify the corrections numerically. As we know the top quark, like no other quark, is accessible to perturbative predictions by virtue of its very large mass and small width, which let the top quark decay before it can form hadrons.

Since free quarks are not observable in nature, their masses primarily are Lagrangian parameters which parametrize the chiral symmetry breaking in terms of masses as required by observation, mainly by the observed mass spectrum of the hadronic states, which consist of permanently confined quarks and gluons. In any case, quark masses are needed as input parameters for calculations of SM predictions [5] and must be tuned to account for corresponding mass effects in hadronic reactions. The most frequently used definitions of masses are the pole and $\overline{\text{MS}}$ ones, which for quarks both are formal definitions. They both are popular because of their simple access in perturbation theory. One should note that the $\overline{\text{MS}}$ scheme is intrinsically only defined in the perturbative approach.

Applying dimensional regularization [6] and the $\varepsilon = (4 - d)/2$ expansion, the RG functions are uniquely defined, order by order in perturbation theory, by the ultraviolet (UV) properties of the model, represented by the $1/\varepsilon$ counterterms [7]. In order to determine the value of a running mass at some scale, the matching condition between the running mass and some observable has to be evaluated (see e.g. Ref. [8]). Since the SM includes both, EW and QCD type UV singularities, the corresponding RG equations have to take into account both, too.

The pole mass is a well defined quantity within perturbation theory. It is related to the pole of the renormalized propagator in the complex energy plane. The position of the pole is a gauge invariant and infrared finite quantity [9, 10]. A shortcoming is the fact that the pole mass suffers from renormalon contributions, which worsen the convergence properties of the perturbative expansion. The corresponding uncertainty is of the order of $\Lambda_{QCD} \sim 200$ MeV [11, 12], not too large for a particle as heavy as the top quark, but it leads to an intrinsic limitation of the possible precision. The top quark being a colored object, the pole of its propagator is not an observable per se, although it seems that the color singlet recombination via gluonic strong-interaction effects does not affect the location of the top quark propagator pole very much. These problems and deficiencies have triggered many discussions about the accuracy of the top quark mass and its extraction from experimental data, and actually other mass definitions which look to be closer to observable quantities have been worked out [13, 14, 15]. Usually, alternative masses are nevertheless converted into pole and/or $\overline{\text{MS}}$

masses, which thus both remain useful concepts, and their relationship remains of primary interest. However, up to now, only QCD corrections have been included in the conversion between pole and $\overline{\text{MS}}$ masses of the top quark. In this note we will discuss how to account for the EW contributions and evaluate their size.

We will denote a pole mass by capital M and a $\overline{\text{MS}}$ mass by lowercase m in the following.

2 The running masses in the Standard Model

The first systematic inclusion of the EW corrections into the definition of the running mass of a fermion has been achieved in Ref. [1]. By including all selfenergy diagrams (including tadpoles), one obtains a gauge invariant relation between pole and bare masses [16]. By applying minimal subtraction to the UV counterterms of this relation, the one-loop relation between a $\overline{\text{MS}}$ mass m_f and the corresponding pole mass M_f , as well as the threshold relation $\delta_{f,\alpha}$ between the corresponding Yukawa coupling $y_f(\mu^2)$ and M_f , have been derived. In this approach, care has to be exercised, especially at the multiloop level, to include all the contributing diagrams including tadpoles, while it is not sufficient to select gauge invariant subsets. As an illustrative example, we mention the $O(\alpha\alpha_s)$ mixing contributions to the pole mass of quarks. The definition, via a “gauge invariant set of diagrams including tadpole contributions”, was complement in Ref. [17] by a theorem about the interrelation between the RG functions for the massive parameters (masses of particles, as well as the Fermi constant) calculated in the broken phase of the SM with RG functions of parameters of the unbroken phase of the SM, in accord with the expectation that spontaneous symmetry breaking does not affect the UV structure of the SM. In other words, the EW UV counterterms in the broken phase of the SM can be obtained in terms of the UV counterterms in the unbroken phase.¹ The mentioned theorem has been verified explicitly by a two-loop analysis of the UV counterterms evaluated in the broken phase of SM [2, 17]. This approach gives rise to the same set of quark self-energy Feynman diagrams [18] as well as to an equivalent definition [19] of the threshold relations [1, 20].

Before we proceed, let us remind the reader of some basic definitions needed for the discussion. Applying dimensional regularization [6] in the broken phase, the SM UV counterterms for the quark masses in the $\overline{\text{MS}}$ scheme have the following form:

$$m_{q,\text{bare}} = m_q(\mu^2) \left[1 + \alpha_s \sum_{i=0} \alpha_s^i \sum_{k=1}^{i+1} \frac{\delta Z_{\alpha_s}^{(i,k)}}{\varepsilon^k} + \alpha \sum_{i,j=0} \alpha^i \alpha_s^j \sum_{k=1}^{i+j+1} \frac{\delta Z_{\alpha}^{(i,j,k)}}{\varepsilon^k} \right]. \quad (1)$$

The first series in this relation corresponds to the QCD corrections, the second one to the EW contribution mixed in higher orders with QCD. In accordance with 't Hooft's

¹A different theorem states that tadpole terms, which are absent in the symmetric phase, drop out from observable quantities. However, if one omits tadpole terms in relations between bare and renormalized quantities, as frequently done in SM calculations, one not only loses a manifestly gauge invariant relationship between the bare and the renormalized theory, also the UV structure is not preserved and one does not get the same RG equations. In fact, tadpoles are related to quadratic divergences which show up in the renormalization of the mass parameter m^2 of the Higgs potential in the symmetric phase.

prescription [7], the quark mass anomalous dimension, defined by

$$\mu^2 \frac{d}{d\mu^2} \ln m_q^2 = \gamma_q(\alpha_s, \alpha) = \left[\alpha_s \frac{\partial}{\partial \alpha_s} + \alpha \frac{\partial}{\partial \alpha} \right] \left[\alpha_s \sum_{i=0} \alpha_s^i \delta Z_{\alpha_s}^{(i,1)} + \alpha \sum_{i,j=0} \alpha^i \alpha_s^j \delta Z_{\alpha}^{(i,j,1)} \right], \quad (2)$$

can be split into two parts: the QCD and the EW contributions $\gamma_q(\alpha_s, \alpha) = \gamma_q^{QCD} + \gamma_q^{EW}$, where γ_q^{QCD} includes all terms which are proportional to powers of α_s only and γ_q^{EW} includes all other terms proportional to at least one power of α , and beyond one loop multiplied by further powers of α and/or α_s . We call γ_t^{QCD} the QCD anomalous dimension and γ_t^{EW} the EW one. As has been shown in Ref. [17], the EW contribution to the fermion anomalous dimension γ_t^{EW} in the $\overline{\text{MS}}$ scheme may be written in terms of RG functions of parameters in the unbroken phase of the SM:

$$\gamma_t^{EW} = \gamma_{y_t} + \frac{1}{2} \gamma_{m^2} - \frac{1}{2} \frac{\beta_\lambda}{\lambda}, \quad (3)$$

where $\gamma_{m^2} \equiv \mu^2 \frac{d}{d\mu^2} \ln m^2$, $\beta_\lambda \equiv \mu^2 \frac{d}{d\mu^2} \lambda$, $\gamma_{y_q} \equiv \mu^2 \frac{d}{d\mu^2} \ln y_q$, y_q is the quark Yukawa coupling, and m^2 and λ are the parameters of the scalar potential V : $V = \frac{m^2}{2} \phi^2 + \frac{\lambda}{24} \phi^4$.

It has been shown also (see Ref. [21]) that the coefficients of the higher poles in ε in the mass counterterms (1) in the broken phase are uniquely determined by the lower-order coefficients and the RG functions defined by Eq. (3).

The RG equation for the squared vacuum expectation value $v^2(\mu^2)$ follows from the RG equations for masses and massless coupling constants:

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} v^2(\mu^2) &= 4 \mu^2 \frac{d}{d\mu^2} \left[\frac{m_W^2(\mu^2)}{g^2(\mu^2)} \right] = 4 \mu^2 \frac{d}{d\mu^2} \left[\frac{m_Z^2(\mu^2) - m_W^2(\mu^2)}{g'^2(\mu^2)} \right] \\ &= 3 \mu^2 \frac{d}{d\mu^2} \left[\frac{m_H^2(\mu^2)}{\lambda(\mu^2)} \right] = 2 \mu^2 \frac{d}{d\mu^2} \left[\frac{m_f^2(\mu^2)}{y_f^2(\mu^2)} \right] \equiv v^2(\mu^2) \left[\gamma_{m^2} - \frac{\beta_\lambda}{\lambda} \right], \end{aligned} \quad (4)$$

where g' and g are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, respectively, and we assume that the running of g, g' as well as y_t and λ are the same in the broken and the unbroken phase (see Refs. [19, 22, 23, 24, 25, 26, 27, 28]).

We note that the anomalous dimension of $v^2(\mu^2)$ defined by Eq. (4) via diagrammatic calculations is not equal to the anomalous dimension of the scalar field as it is obtained in the effective-potential approach [29].

The RG equations (2) have to be complemented by matching conditions between pole and running masses, which we may write in the form

$$M_t - m_t(\mu^2) = m_t(\mu^2) \sum_{j=1} \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^j \rho_j + m_t(\mu^2) \sum_{i=1; j=0} \left(\frac{\alpha(\mu^2)}{\pi} \right)^i \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^j r_{ij}. \quad (5)$$

The QCD corrections ρ_j have been calculated in Refs. [30, 31, 32] up to $j = 3$, while for the EW part, the $O(\alpha)$ and $O(\alpha\alpha_s)$ corrections r_{10} and r_{11} , respectively, are available [1, 2, 16]. The correction r_{12} has been evaluated in Ref. [33] in the gaugeless-limit approximation of the SM.

3 The behavior of the RG equations at low and high energies

Let us analyze the behavior of the full SM RG equation for a quark mass in the $\overline{\text{MS}}$ scheme

$$\mu^2 \frac{d}{d\mu^2} \ln m_f(\mu^2) = \gamma_q^{QCD} + \gamma_{y_f} - \frac{1}{2} \gamma_{G_F} . \quad (6)$$

Let us consider the low-energy limit first. In the weak sector of the SM there is no decoupling because masses and couplings are interrelated by the Higgs mechanism. So “decoupling by hand” as usually applied in QCD by considering an effective ‘ n_f active flavors’ QCD to be matched at successive flavor thresholds, and which can be applied to QED as well, does not make sense in the weak sector. Note that there is no decoupling for the W and Z bosons: the limit $M_W \rightarrow \infty$ can be achieved by letting $g \rightarrow \infty$ or $v \rightarrow \infty$ or both. In nature, only the limit $g \rightarrow \infty$ leads to the observed low-energy limit of the effective four-fermion theory with $\sqrt{2} G_\mu = 1/v^2$ fixed, by nuclear β -decay etc. This obviously is a non-decoupling effect. In contrast to QED or QCD, the low-energy effective theory (obtained after elimination of the heavy state) is a non-renormalizable one exhibiting a completely wrong high-energy behavior.

In this way, for low values of the energy scale μ , when $\mu < M_t, M_H, M_Z, M_W$, the bosonic and top-quark terms on the r.h.s. of Eq. (6) are equal to zero while the light-fermion contributions proportional to $G_F m_f^2$ are tiny. Consequently, below the W boson mass, the effective Fermi constant practically does not change with scale. In this case, below of the EW scale the natural form of the EW contribution to the ratio between pole and running masses is

$$m_f^{EW}(\mu^2) = 2^{-3/4} G_F^{-1/2} y_f^{\text{eff}}(\mu^2) , \quad (7)$$

where only light particles contribute to $y_f^{\text{eff}}(\mu^2)$. A well-known remarkable property of the Yukawa coupling $y_f(\mu^2)$ is that EW radiative corrections to it are free of tadpoles [1, 18] and/or quadratic divergences. Since real physical observables are also free of tadpole contributions, this property is an additional argument why Eq. (7) is a good candidate for the evaluation of the EW contributions to the ratio between pole and $\overline{\text{MS}}$ masses of lighter quarks, such as the bottom and charm quarks (see also the discussion in Ref. [34]). In short: fermion masses and Yukawa couplings have equivalent RG evolution as long as G_F or, equivalently, the Higgs vacuum expectation value (VEV) v are not running and $G_F^{\overline{\text{MS}}}(\mu) = G_F$.

The running of G_F only starts at about $\mu \sim 2 M_W$, when the scale of a process exceeds the masses of the weak gauge bosons. However, since the top quark is the heaviest particle in the SM, we do not see evidence, why the “decoupling by hand” prescription should be applied to the top quark.

One of the famous non-decoupling effects related to the top quark is the EW ρ parameter: $\rho_{\text{eff}}(0) = G_{\text{NC}}/G_{\text{CC}}(0)$ where $G_{\text{CC}}(0)$ is the Fermi coupling $G_F = G_\mu$ and G_{NC} the low-energy effective axial vector Z coupling to fermions. As is well known, in the SM we have $\rho = 1 + \frac{\sqrt{2} G_\mu}{16\pi^2} N_c |m_t^2 - m_b^2| = 1 + \frac{1}{32\pi^2} N_c |y_t^2 - y_b^2|$, which measures the weak-isospin breaking by the Yukawa couplings of the heavy fermions at zero

momentum. Within the SM, this quantity is strongly constrained by LEP data, and, in spite of the fact that the top quark was by far too heavy to be produced at LEP, the top quark contribution and indirectly the top quark mass have been constrained by LEP. Actually, a first strong indication of a heavy top quark had been found earlier by the ARGUS experiment, which discovered, unexpectedly, a substantial $B\bar{B}$ oscillation (in the SM enhanced by a contribution $\propto y_t^2$), which turned out to be much larger than anticipated before. So recipes like “decoupling by hand” make no sense to be applied to the weak sector, as heavy-particle effects definitely cannot be renormalized away.

For large values of μ^2 , the behavior of the running Fermi constant $G_F(\mu^2)$ is defined by the Higgs self-coupling and the sign of its beta-function β_λ :

$$\mu^2 \frac{d}{d\mu^2} \ln G_F(\mu^2) \sim \frac{\beta_\lambda(\mu)}{\lambda(\mu)}.$$

The detailed perturbative analysis of the r.h.s. of this equation was performed recently (see Refs. [19, 22, 23, 24, 26, 27]) and reveals that the beta function β_λ is negative up to a scale of about 10^{17} GeV, where it changes sign. Above the zero of β_λ , the effective coupling starts to increase again, and the key question is whether at the zero of the beta function the effective coupling is still positive. In the latter case, it will remain positive although small up to the Planck scale. In any case, at moderately high scales where $\beta_\lambda < 0$, and provided that λ is still positive, the following behavior is valid for the Fermi constant:

$$G_F(\mu^2) \Big|_{\mu^2 \rightarrow \infty} \sim \left(\mu^2 \right)^{\frac{\beta_\lambda(\mu)}{\lambda(\mu)}} \rightarrow 0, \quad (8)$$

being decreasing, which means $v^2(\mu^2)$ is increasing at these scales (where $\beta_\lambda < 0$ and $\lambda > 0$). The analysis of Ref. [22] finds that λ turns negative (unstable or meta-stable Higgs potential) before the beta function reaches its zero. This may happen at rather low scales around 10^{10} GeV. In this case, we would get an infinite Higgs vacuum expectation value far below the Planck scale as an essential singularity. We find it more likely that λ remains positive up to the zero of the beta function and as a consequence up to the Planck scale [19, 23]. Then $G_F(\mu^2)$ would start to increase again, and $v(\mu^2)$ would start to decrease but remain finite (about 685 GeV) at the Planck scale, implying that all effective masses stay bounded. The effectively massless symmetric phase of the SM would then be obtained at high energies by the fact that mass effects are suppressed for dimensional reasons: according to the RG, for a vertex function under a dilatation of all momenta, $\{p_i\} \rightarrow \{\kappa p_i\}$, up to the overall dynamical dimension and wave function renormalizations the result is given by replacing $g_i \rightarrow g_i(\kappa)$ and $m_i \rightarrow m_i(\kappa)/\kappa$ at fixed $\{p_i\}$ and renormalization scale μ , i.e. provided that $m(\kappa)/\kappa \rightarrow 0$ as $\kappa \rightarrow \infty$, the high-energy asymptotic effective theory is effectively massless as expected in the symmetric phase.

4 Numerical results for $m_t - M_t$

In the previous section, we have presented the arguments, why decoupling does not apply in the EW sector, in particular not to the top quark mass effects. In this section, we will check how significant the EW contribution to matching and running of the top

quark mass is. For that purpose, the inverse of the relation (5), $m_t(\mu^2)$ as a function of the pole mass M_t , is required (see Eq. (5.54) in the first paper of Ref. [2]). For the numerical analysis, we adopt the following values for the input parameters [35]:

$$\begin{aligned} M_Z &= 91.1876(21) \text{ GeV}, \quad M_W = 80.385(15) \text{ GeV}, \quad M_t = 173.5(1.0) \text{ GeV},^2 \\ G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha^{-1} = 137.035999, \quad \alpha_s(M_Z^2) = 0.1184(7). \end{aligned} \quad (9)$$

Furthermore, we take the effective fine-structure constant at the Z boson mass scale to be $\alpha^{-1}(M_Z^2) = 127.944$. All light-fermion masses M_f ($f \neq t$) give negligible effects and do not play any role in our consideration. For our purpose, it suffices to include the one-loop running of the strong-coupling and of the electromagnetic fine-structure constant. For scales above the W boson mass, but below the top quark mass scale, we then have

$$\alpha_s(\mu^2) = \frac{\alpha_s(M_Z)}{1 + \frac{23}{12} \frac{\alpha_s(M_Z)}{\pi} \ln \frac{\mu^2}{M_Z^2}}, \quad \alpha(\mu^2) = \frac{\alpha(M_Z)}{1 - \frac{11}{12} \frac{\alpha(M_Z)}{\pi} \ln \frac{\mu^2}{M_Z^2}}. \quad (10)$$

Up to the three-loop order, the QCD relation between the running and pole masses is given by (see Eq. (12) in Ref. [32])

$$\{m_t(M_t) - M_t\}_{QCD} = M_t \left\{ -\frac{4}{3} \frac{\alpha_s}{\pi} - 9.125 \left(\frac{\alpha_s}{\pi} \right)^2 - 80.405 \left(\frac{\alpha_s}{\pi} \right)^3 \right\}. \quad (11)$$

Taking into account the running of α_s from the scale M_Z to M_t , where it suffices to use the one-loop approximation given above, we obtain the numerical result

$$\{m_t(M_t) - M_t\}_{QCD} = -7.98 \text{ GeV} - 1.88 \text{ GeV} - 0.57 \text{ GeV} = -10.43 \text{ GeV}. \quad (12)$$

A numerical estimation of the $O(\alpha_s^4)$ term, given in Ref. [36], is $\sim -0.02 \text{ GeV}$, which is not included in Eq. (12). The analytic result for the EW corrections at the one-loop order has a more complicated form and may be found in Refs. [1, 16]. The two-loop corrections of order $O(\alpha^2)$ are not yet known. Exploring the results of Ref. [33], we estimate it to be of order $O(1 \text{ GeV})$. Another way to estimate the two-loop contribution follows from results of Ref. [17] and the observation that the largest contribution is coming from tadpole diagrams:

$$\left\{ \frac{m_t(\mu^2)}{M_t} \right\} \sim \left\{ \frac{m_t(\mu^2)}{M_t} \right\}_{\text{tadpole}} \sim \sqrt{\frac{m_W^2(\mu^2)}{M_W^2}} = \left(1 + \frac{1}{2} \delta_{W,\alpha} + \frac{1}{2} \delta_{W,\alpha^2} - \frac{1}{8} \delta_{W,\alpha}^2 \right), \quad (13)$$

where we put (for details, see Ref. [17]) $\frac{m_W^2(\mu^2)}{M_W^2} = 1 + \delta_{W,\alpha} + \delta_{W,\alpha^2} + \delta_{W,\alpha s \alpha}$. This also allows us to estimate the error due to the unknown higher-order corrections, which is about 1 GeV.

²The values of the top quark mass quoted by the experimental collaborations correspond to parameters in Monte Carlo event generators in which the partonic subprocesses are treated at the tree level, so that a rigorous theoretical definition of the top quark mass is lacking. For definiteness, we take the value from Ref. [35] to be the pole mass.

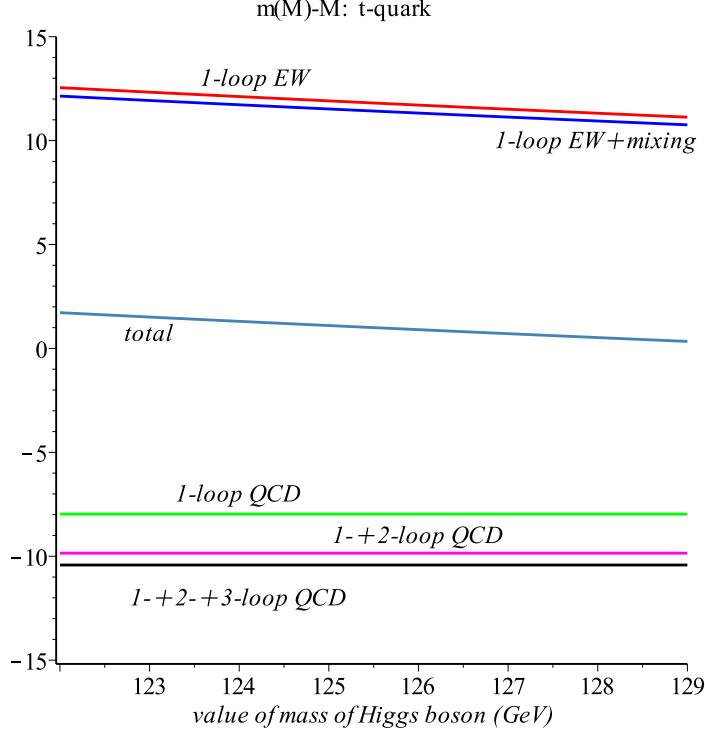


Figure 1: Numerical results for the difference $m_t(M_t) - M_t$. The red line represents the $O(\alpha)$ correction, the blue line the $O(\alpha) + O(\alpha\alpha_s)$ correction, the green line the $O(\alpha_s)$ correction, the magenta line the $O(\alpha_s) + O(\alpha_s^2)$ correction, the black line the $O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3)$ correction, and the light blue line the sum of all these corrections. The input parameters are given in Eq. (9).

A detailed comparison of the individual contributions is presented in Fig. 1. For a set of experimentally most probable values of the Higgs boson mass [3, 4], $M_H = \{124, 125, 126\}$ GeV, the numerical values of the EW contribution to the difference $m_t(M_t) - M_t$ and the total result, the sum of QCD and EW corrections, are collected in Tab. 1.

Table 1: The various contributions to $m_t(M_t) - M_t$.

$m_t(M_t) - M_t$ [GeV]				
M_H [GeV]	$O(\alpha)$	$O(\alpha\alpha_s)$	$O(\alpha) + O(\alpha\alpha_s)$	total
124	12.11	-0.394	11.7244	1.303
125	11.91	-0.389	11.5224	1.101
126	11.71	-0.384	11.3251	0.904

As a result, we observe a surprisingly large EW correction, which for the known Higgs boson mass range almost perfectly cancels the QCD correction. The relationship

between $m_t(M_t)$ and M_t can be parametrized in the range displayed in Fig. 1 as

$$\{m_t(M_t) - M_t\}_{SM} = \{m_t(M_t) - M_t\}_{QCD} + \left[0.0664 - 0.00114 \times \left(\frac{M_H}{1 \text{ GeV}} - 125 \right) \right] M_t. \quad (14)$$

The almost perfect cancellation between the QCD and EW effects for the given Higgs boson mass is certainly accidental, but must be taken into account in comparisons with data.

5 Conclusion

We have analyzed the EW contributions to the running and scheme dependence of the top quark mass above the W boson threshold, when G_F can not be treated any longer as a low-energy constant in one-to-one correspondence with the muon lifetime, but turns into a running effective parameter. This effect is similar to the running electromagnetic coupling $\alpha(\mu^2)$, which, however, is strongly scale dependent right from zero momentum and is sensitive to non-perturbative hadronic vacuum polarization effects there. Like the running couplings g, g', y_f , and λ , also the running of G_F is scheme dependent. In the $\overline{\text{MS}}$ scheme, the scale at which G_F effectively starts to run, is not uniquely defined. SM non-decoupling effects have to be taken into account. In any case, light-fermion contributions including the one of the bottom quark are tiny. The quantitative analysis shows that the main contribution comes from the matching relation (5), which supplements the RG equation (6). At low energies, the running of the quark mass is equivalent to the running of the Yukawa coupling via Eq. (7) and by standard QCD corrections.

As the $\overline{\text{MS}}$ scheme is a mass-independent renormalization scheme, mass effects drop out at high energies on account of their positive canonical mass dimension. This is in contrast to on-shell renormalization schemes, where masses are utilized as renormalization scales, which leads to residual mass effects in the high-energy asymptotic regime via renormalization effects (Callan-Symanzik equation replacing the $\overline{\text{MS}}$ RG equations).

As our focus is on physics at the EW scale, a precise treatment of mass effects of the heavier SM states (t, H, Z, W) is mandatory for a precise interpretation of related experimental data. In particular, for the top quark (which as we know decays before it can form hadrons) it is not sufficient to take into account QCD corrections only, as our analysis shows.

In conclusion, for the current value of the Higgs mass, $122 < M_H < 128 \text{ GeV}$ [3, 4], the one-loop EW corrections are large and have opposite sign relative to the QCD contributions, so that the total correction is actually small and approximately equal to $1 \pm O(1) \text{ GeV}$ (see Table I). The value of $m_t(m_t)$ can be obtained by iteration of Eq. (5) starting from $m_t(M_t)$, with the result $m_t(m_t) \sim M_t \pm O(3) \text{ GeV}$. As a result, taking into account EW radiative corrections, besides the QCD ones, reduces the scheme dependence for EW observables, which depend on the top quark mass.

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